

# Problem A Archaeological Collection Mystery Time limit: 5 seconds

#### Mille, Ethiopia

In the highlands of Ethiopia, the Homo Sapiens Paleoanthropology Committee (HSPC) has made a remarkable discovery at a previously unexplored dig site. They've uncovered numerous human skeletal remains dating back thousands of years, but the bones are scattered and fragmented throughout the excavation area.

You've been tasked with determining the possible number of individuals represented in this collection. You know that a complete human skeleton contains a specific number of each type of bone (for example, 1 skull, 2 femurs, 12 ribs, etc.). However, due to natural decomposition, animal scavenging, and other taphonomic processes, not all bones from each individual have survived.

Given the number of each type of bone found at the excavation  $(v_i)$  and the number of each bone type in a complete human skeleton  $(h_i)$ , you need to calculate both the minimum and maximum number of individuals that could be represented in the remains.



Ethiopia. By Radosław Botev, CC BY 3.0 pl, acquired from Wikimedia Commons.

#### Input

The first line contains a single integer  $n \ (1 \le n \le 10^5)$ -the number of different

bone types considered in the analysis. The second line contains n space-separated integers  $h_1, h_2, \ldots, h_n$   $(1 \le h_i \le 10^3)$ -the number of each bone type in a complete human skeleton. The third line contains n space-separated integers  $v_1, v_2, \ldots, v_n$   $(0 \le v_i \le 10^3)$ -the number of each bone type found at the excavation site.

### Output

Output two space-separated integers-the minimum and maximum possible number of individuals represented by the remains.

Sample Input 1	Sample Output 1
3	5 15
1 2 2	
5 7 3	

Samp	ole Ir	nput	2
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#### Sample Output 2

4	9 123
1 3 2 26	
5 27 16 75	



Sample Input 3	Sample Output 3
1	1 1
2	
1	
Sample Input 4	Sample Output 4
Sample Input 4	Sample Output 4
Sample Input 4	Sample Output 4



# Problem B Baursaks Time limit: 5 seconds

Astana, Kazakhstan

Nick, a passionate foodie, has traveled to the vast steppes of Kazakhstan in search of legendary baursaks (a type of traditional Kazakh fried dough). After days of research, he's mapped their locations along a 1-dimensional plain. Starving from his journey, Nick must optimize his path to consume as many Calories as possible.

There are n baursaks placed at various integer positions along a line. Each position can have at most one baursak. Nick starts at position 0 with 0 Calories. Moving 1 unit of distance (either left or right) costs 1 Calorie. Nick can never have negative Calories–if Nick doesn't have enough Calories to make a move, he cannot move. Each baursak contains  $c_i$  Calories that Nick can consume when he reaches its position.



A wild baursak carrying its young

Nick's goal is to consume as many Calories as possible. Determine how many Calories he can consume.

#### Input

The first line contains a single integer  $n \ (1 \le n \le 2000)$ -the number of baursaks.

The next n lines each contain two integers  $p_i$  and  $c_i$  ( $-10^8 \le p_i \le 10^8$ ,  $1 \le c_i \le 10^6$ )-the position of the *i*<sup>th</sup> baursak on the number line and the amount of Calories it contains.

All  $p_i$  values are guaranteed to be distinct, but they are not necessarily given in sorted order.

#### Output

Output a single integer-the maximum amount of Calories Nick can consume.

Sample Input 1	Sample Output 1
2	1
0 1	
2 3	

#### Sample Input 2 Sample Output 2

4	11
0 2	
-2 5	
-3 1	
3 4	



# Problem C Nile Time limit: 5 seconds

Cairo, Egypt

Oh no-ACM has gotten lost on the Nile River! Help them figure out how likely they are to get to each city.

The Nile River flows south to north and has a single entrance<sup>1</sup>. Along the way, it passes through some cities. At each city, ACM has some chance of making it to the shore, or continuing to some city immediately downstream. Each city has exactly one city immediately upstream of it.

For each city, what is the probability that ACM makes it to the shore in that city?

#### Input

The first line contains the integer N ( $1 \le N \le 10^5$ ), the number of cities. City #1 represents the entrance to the Nile.

The  $k^{\text{th}}$   $(1 \le k \le N - 1)$  line of the next N - 1 lines contains two space-separated values  $j_{k+1}, p_{k+1}$   $(1 \le j_{k+1} \le k \text{ and } 0 < p_{k+1} \le 1)$  where  $j_{k+1}$  is an integer and  $p_{k+1}$  has at most 6 places after the decimal. This represents the probability of going from city  $j_{k+1}$  to city k + 1, given that we are currently at  $j_{k+1}$ .

It is guaranteed that  $\sum_{\substack{2 \le k \le N \\ j_k = i}} p_k \le 1$  for all *i*, where the sum is over all nodes with parent *i*.

### Output

Output N lines, where the  $k^{\text{th}}$  line  $(1 \le k \le N)$  represents the probability of ending up at city k.

For all probabilities, your answer will be accepted if the absolute or relative error is at most  $10^{-6}$ . That is, if the correct answer is y and your answer is x, your answer will be accepted if  $\min\left(|x-y|, \frac{|x-y|}{y}\right) \le 10^{-6}$ .

Sample Input 1	Sample Output 1
5	0.44003499999999995
1 0.559965	0.247481571435
2 0.558041	0.10189865867104655
3 0.673907	0.011819491379837926
4 0.943873	0.19876527851411557

Sample Input 2	Sample Output 2
8	0.0572179999999995
1 0.844215	0.844215
1 0.098567	0.012113588598999997
3 0.877103	0.05074313819452575
4 0.413058	0.01881620718604297
5 0.30143	0.010764147652627534
5 0.171657	0.0034480361724610357
7 0.437507	0.002681882195342716

<sup>1</sup>At least, as far as the ACM is aware.



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Sample Input 3	Sample Output 3	
12	0.4798389999999996	
1 0.022149	0.01417447404	
2 0.36004	0.00797452596	
1 0.193895	0.132841536295	
4 0.314879	0.04967456335351693	
5 0.186376	0.0016250662747967042	
6 0.857186	0.009753834076686375	
1 0.004507	0.002849203711	
8 0.367827	0.0008188717925441279	
9 0.506048	0.0008389244964558721	
1 0.29961	0.24531257853	
11 0.181227	0.054297421469999996	



# Problem D National Museum of Korea Time limit: 5 seconds

#### Seoul, South Korea

The Hanguk Society for the Preservation of Culture (HSPC) is organizing an event for high schoolers in the east wing of the National Museum of Korea. As high schoolers can be rowdy at times, the HSPC would like to place chaperones in some of the exhibit rooms to ensure that no priceless artifacts get damaged. The chaperones may guard up to two rooms at a time, as long as the door between them is not closed. However, it's not yet certain which doors between exhibit rooms will remain open at the event. The HSPC would like to know how many volunteer chaperones it needs, even as the door plans change.

The east wing can be modeled as a  $2 \times n$  grid, where each grid cell represents a room containing an exhibit. The boundaries between grid cells represent doors. The doors are numbered.

- The n-1 doors between adjacent rooms on the top row are numbered 1 through n-1, from left to right.
- The *n* doors between rooms in the top and bottom rows of the grid are numbered *n* through 2n 1, from left to right.
- The n-1 doors between adjacent rooms on the bottom row are numbered 2n through 3n-2, from left to right.

See the diagram for more information.



Chaperones may guard one room, or they may guard two adjacent rooms that are separated by a single open door. Initially, all doors are closed. As doors open and close, find (a)  $c_i$ , the minimum number of chaperones needed to guard every room and (b) the number of ways to divide up the rooms into  $c_i$  groups so that they may be guarded by  $c_i$  chaperones.

Note that there is one way to guard a pair of rooms with an open door between them by using a single chaperone (by placing a chaperone in charge of both rooms). On the same note, if the door were closed, there would still be only one way to guard those rooms by using two chaperones (by placing a chaperone in charge of each room).

#### Input

On the first line are two integers n, q, the width of the museum and the number of door toggles ( $2 \le n \le 100\,000$ ,  $1 \le q \le 100\,000$ ).

The following q lines each contain a single integer  $d_i$ , the index of the door to toggle.  $(1 \le d_i \le 3n - 2)$ .

## Output

Output q lines. On the  $i^{\text{th}}$  line, output two integers  $c_i$ ,  $w_i$ , where  $c_i$  is the number of chaperones needed after the first i toggles. Since it could be large, let  $w_i$  be the remainder when the number of ways to use those chaperones is divided by 998 244 353.



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Sample Input 1	Sample Output 1
2 5	3 1
1	3 2
2	2 1
3	2 2
4	2 1
1	



# Problem E ¡Pura Vida! Time limit: 5 seconds

San José, Costa Rica

You've tried to send a postcard home to your family during your trip to Costa Rica, but the Horrible Seaborne Postal Company (HSPC) has misplaced it! You've managed to capture a blurry picture of the postcards in the lost-and-found-you can't make out all the letters, but you know that yours said "¡Pura Vida!" on the front.

After recording the letters from the image (and writing down a \* where you weren't sure what the letter was), you've set out to find your postcard.

Given the partially-blurred sequence of letters in the image, where is the first location that the string PURAVIDA could be?

#### Input

The input consists of a single string s ( $1 \le |s| \le 100$ ) consisting only of the upper case Latin letters A-Z and asterisks \*.

### Output

Print the first index in s (starting from 1) where the string PURAVIDA could be. If no such index exists, print -1.

Sample Input 1	Sample Output 1
P*RAV*DAPURAVIDA	1
Sample Input 2	Sample Output 2
ABCDEFGHIJKLM	-1
Sample Input 3	Sample Output 3
BIGDAYHSPURAVIDCJPURAVIDAABC	18



# Problem F Turkish Delight Time limit: 5 seconds

Ankara, Turkey

A Turkish delight vendor is organizing a display at a bazaar in Ankara. Turkish delights are beautiful candies which come in many flavors. Passers-by will add and remove candies from the display of candies as the day goes on. The vendor would like to know the number of different flavors remaining each time.

There is initially a horizontal line of candies on display. Passers-by can add a candy to either end of the line (either the left end or the right end). But they may also be greedy, and eat the first candy they can as they're passing by (either from the left end or the right end).



Some small Turkish delights. By Appaloosa - Self-photographed, CC BY-SA 3.0, acquired from Wikimedia Commons.

After each passerby comes and goes, report the number of different flavors remaining.

### Input

The first line contains two space-separated integers n and c, the initial number of candies and the number of flavors  $(1 \le n, c \le 10^5)$ . Each flavor is an integer from 1 to c (inclusive).

The second line contains n space-separated integers  $a_1, \ldots, a_n$   $(1 \le a_1, \ldots, a_n \le c)$ , indicating the initial flavors on display.

The third line contains one integer q ( $1 \le q \le 10^5$ ) denoting the number of passers-by.

Each of the next q lines denotes an action of a passerby, where d is either L or R, representing whether the action is on the left or right side, respectively.

- E d
- A d k

E means a passerby eats the candy on that end, while A means they add a Turkish delight of type k ( $1 \le k \le c$ ).

## Output

Output q lines, where the  $i^{th}$  line represents the number of unique flavors present after the  $i^{th}$  person passes by.

Sample Input 1	Sample Output 1
5 3	3
1 3 2 2 3	2
4	2
ER	3
EL	
AL3	
AL1	



Sample Input 2	Sample Output 2
5 7	3
1 3 2 2 3	3
4	2
ER	1
ER	
ER	
ER	



# Problem G Seeds Time limit: 5 seconds

Longyearbyen, Norway

The Horticultural Seed Preservation Committee (HSPC) is dedicated to preserving the seeds of all varieties of plants. They are planning on building a seed vault that rivals the size of the Svalbard Global Seed Vault. For security and exposure from the elements, they'd like it be in a mountain, and they've asked you to find out where it should go.

You are given the elevation of the mountain (in meters) at every point and must find the largest volume of an axis-aligned rectangular prism that fits inside the mountain.



Exterior of Svalbard Global Seed Vault. By Cierra Martin for Crop Trust - Flickr, CC BY 2.0, acquired from Wikimedia Commons.

### Input

The first line contains two space-separated integers  $h, w (1 \le h, w \le 500)$  denoting the height and width, respectively, of the mountain's base.

Each of the next h lines contains w space-separated integers, where each is between 0 and  $10^9$  (inclusive) and denotes the elevation at the corresponding point.

## Output

On one line, output the maximum volume of an axis-aligned rectangular prism that fits beneath the mountain.

Sample Input 1	Sample Output 1
3 3	50
1 1 1	
1 50 1	
1 1 1	

Sample Input 2	Sample Output 2
3 5	24
3 3 3 2 2	
2 2 23 2 1	
2 2 2 2 2 2	



# Problem H Kiwi Counting Time limit: 1 second

Wellington, New Zealand

Due to a series of unfortunate mix-ups, the Actinidia Counting Machine (ACM), the Apteryx Counting Machine (ACM), and the Aucklander Counting Machine (ACM) are all under maintenance. In the meantime, you have to count the fruit, the birds, and the New Zealanders yourself!

Fortunately, you've found out a few things about the population of kiwis you're counting. You know that

- Kiwis (the fruit) have no legs and no beaks each. The kiwis you are studying weigh 100 grams each.
- Kiwis (the bird) have two legs and one beak each. The kiwis you are studying weigh 2800 grams each.
- Kiwis (the humans) have two legs and no beaks each. The kiwis you are studying weigh 72000 grams each.

You've recorded the total number of legs and beaks you've seen. You've also recorded the total weight of all of your kiwis.

How many kiwis do you have?

#### Input

On the first line are three space-separated integers L, B, W ( $0 \le L, B, W \le 1000000$ ) – the number of legs, the number of beaks, and the total weight in grams. It is guaranteed that there is exactly one amount of fruit, birds, and humans that would account for your observation.

### Output

On one line, print the total number of kiwis (that is, the sum of the numbers of fruits, birds, and humans).

Sample Input 1	Sample Output 1
6 2 77900	6
Sample Input 2	Sample Output 2



# Problem I Crown Shyness Time limit: 5 seconds

Buenos Aires, Argentina

*Crown shyness* is a phenomenon where trees avoid touching each other. After a lovely visit to Plaza San Martín in Buenos Aires, the newly-formed Herbal Shyness Planning Committee (HSPC) has been inspired to plant a garden of eucalyptus trees to demonstrate this principle.

The garden layout can be represented as a finite, two-dimensional grid. Initially at time t = 0, trees start out as seedlings that occupy only one cell. At every time t, the trees grow. If some cell adjacent to some tree does not already contain a tree and is not adjacent to any other trees, that tree will expand into that cell at time t + 1.



Trees in Plaza San Martín in Buenos Aires By Dag Peak - Flickr, CC BY 2.0, acquired from Wikimedia Commons.

The HSPC would like to know: at time  $t = 10^{2025}$ , what is the total *leaf exposure* of all trees? The leaf exposure of a tree is the number of cell boundaries adjacent to exactly one cell containing that tree.

Trees cannot expand past the boundary of the grid, but cell boundaries on the boundary of the grid do count for leaf exposure calculations.

### Input

On the first line are two integers h, w, the height and width of the garden  $(1 \le h, w \le 100\,000, w \cdot h \le 100\,000)$ .

The following h lines consist of w characters describing the state of the garden at time t = 0. An empty cell is written with a single E, while a cell containing a seed is written with a single S.

### Output

On one line, print the total leaf exposure of all trees at time  $t = 10^{2025}$ .

Sample Input 1	Sample Output 1
2 2	12
SS	
EE	

Sample Input 2	Sample Output 2
2 2	8
SE	
ES	



Sample Input 3	Sample Output 3
3 7	24
EEEEEE	
SEEEES	
EEEEEE	



# Problem J Taj Mahal Time limit: 5 seconds

#### Agra, India

Agra Cultural Moments (ACM) is organizing tours to the Taj Mahal. They'd like to know the amount of free time they have in their schedule.

Each tour starts at some time and ends at some time. Can you help ACM determine how much time in the schedule has no tours scheduled at all?



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Time starts at t = 0.

### Input

The first line contains two space-separated integers n, T ( $1 \le n \le 10^5$  and  $1 \le T \le 10^9$ ), where n is the number of tours already scheduled, and T is the time the schedule ends.

Each of the next n lines contains two space-separated integers  $s_i, e_i \ (0 \le s_i < e_i \le T)$ , denoting the start and end times for the  $i^{\text{th}}$  tour.

### Output

On one line, output the total time from t = 0 to t = T where no tour is scheduled.

Sample Input 1	Sample Output 1
3 100	8
7 78	
10 99	
33 98	

Sample Input 2	Sample Output 2
7 100	66
64 67	
70 78	
46 52	
94 100	
89 97	
51 58	
91 98	



# Problem K Antarctic Crowning Match Time limit: 3 seconds

McMurdo Station, Antarctica

In the frigid expanse of Antarctica, the annual Antarctic Crowning Match (ACM) determines which penguin will be crowned emperor. The match follows ancient penguin tradition, where contestants engage in a strategic snowball elimination contest.

There are n penguins positioned at distinct points on a 2D plane. When the match begins, each penguin simultaneously throws a snowball at their closest competitor(s). If multiple penguins are at the same minimum distance from a thrower, that penguin throws snowballs at all of those equally close competitors. Any penguin hit by at least one snowball is eliminated from the match.

The process continues with the remaining penguins until either one penguin remains (the winner) or all penguins are eliminated (resulting in no winner). As the official tournament judge, your task is to determine if there will be a winner, and if so, which penguin will be crowned emperor.

#### Input

The first line contains a single integer  $n \ (2 \le n \le 1000)$ –the number of penguins in the match.

The next n lines each contain two integers  $x_i$  and  $y_i$  ( $-10^7 \le x_i, y_i \le 10^7$ )-the coordinates of the *i*<sup>th</sup> penguin on the 2D ice sheet.

All coordinates are guaranteed to be distinct.

## Output

Output one line with the index i  $(1 \le i \le N)$  of the last remaining penguin, in the case of a winner. If there is no winner, output No Emperor instead.

Sample Input 1	Sample Output 1
3	No Emperor
0 0	
2 0	
0 2	

Sample Input 2	Sample Output 2

4	3
4 4	
4 5	
0 0	
5 4	



# Problem L Teacups Time limit: 5 seconds

#### Orlando, USA

All the HSPC volunteers and participants are going to Bisneyworld in Florida to celebrate! They've decided to go on the spinning teacups ride–they love going around and around in circles. But the volunteers realized the participants don't have the problem packets, and so they need to throw it to them on the rides. Can you help them figure out how close they can get to each other so they can deliver the packets?

These rides work by having disks (possibly on other disks). You can get two students closer by rotating disks–when you rotate a disk, all disks and points on inside the disk rotate about the center of the disk as well (see figures L.1 and L.2).

It is guaranteed that the boundaries (i.e. the circles) of the disks do not intersect or touch each other. However, disks could be fully contained in other disks. There are no students on the boundaries of disks (for safety reasons).

You (the ride operator) will respond to queries asking how close you can get two students to each other.



Figure L.1: Original

#### Input

The first line contains two integers T, S ( $1 \le T \le 10^5, 2 \le S \le 10^5$ ), where T is the number of rotating teacups, and S is the number of students.

Each of the next T lines contains three space-separated integers  $x, y (-10^7 \le x, y \le 10^7)$  and  $r (1 \le r \le 10^7)$ , which describes a teacup with center (x, y) and radius r.



Figure L.2: After rotation optimal for  $S_1$  and  $S_2$ 

Each of the next S lines contains two space-separated integers  $x_i, y_i$  ( $-10^7 \le x_i, y_i \le 10^7$ ), where the *i*<sup>th</sup> of these lines ( $1 \le i \le S$ ) has the location ( $x_i, y_i$ ) of the *i*<sup>th</sup> student at the beginning.

The next line contains one integer Q  $(1 \le Q \le 10^5)$  which describes the number of queries.

Each of the next Q lines contains two space-separated integers i, j ( $1 \le i, j \le S$  and  $i \ne j$ ). This represents the pair of students that we are trying to minimize the distance for.

### Output

For each query output a line with a single value, which is the minimum possible distance between the students in the query. Your answer will be accepted if the absolute or relative error is at most  $10^{-6}$ . That is, if the correct answer is y and your answer is x, your answer will be accepted if  $\min\left(|x-y|, \frac{|x-y|}{y}\right) \le 10^{-6}$ .

Sample Input 1	Sample Output 1
3 2	3.17157287525
0 0 4	
0 1 2	
7 0 2	
6 1	
-1 0	
1	
1 2	



## Problem M Two Lines Time limit: 5 seconds

Paris, France

On a recent trip to the Louvre, you encountered a piece of mathematical artwork that you really enjoyed. The beautiful artwork is included below.



The artwork was constructed as follows:

- 1. First, the artist picked some integer n (in this case, n = 3).
- 2. Then, the artist drew two parallel lines.
- 3. The artist then drew n distinct points, placing each point on either of the lines.
- 4. The artist then drew an arrow from each point to its closest neighbor. It is guaranteed that among the points placed, every point had exactly one closest neighbor.
- 5. The artist then labelled each point with the number of arrows pointing to it.

You're wondering about what [multi]sets of labels could be produced by different choices of points. You already know that  $\{0, 1, 2\}$  is possible-how hard could it be?

#### Input

The first line contains a single integer t, the number of sequences to test  $(1 \le t \le 10^5)$ .

Each of the next t pairs of lines describes a sequence. The first line of the pair contains a single integer n ( $2 \le n \le 10^5$ ). The second line of the pair contains n integers  $a_1, \ldots, a_n$  ( $0 \le a_i \le 10^9$ )-the sequence in question.

It is guaranteed that the sum of all n is at most  $10^6$ .

## Output

For each sequence, output YES if the sequence could be made (up to reordering) by following the painting algorithm above. Otherwise, print NO.



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Sample Input 1	Sample Output 1
3	YES
2	YES
1 1	NO
3	
1 2 0	
2	
2025 0	